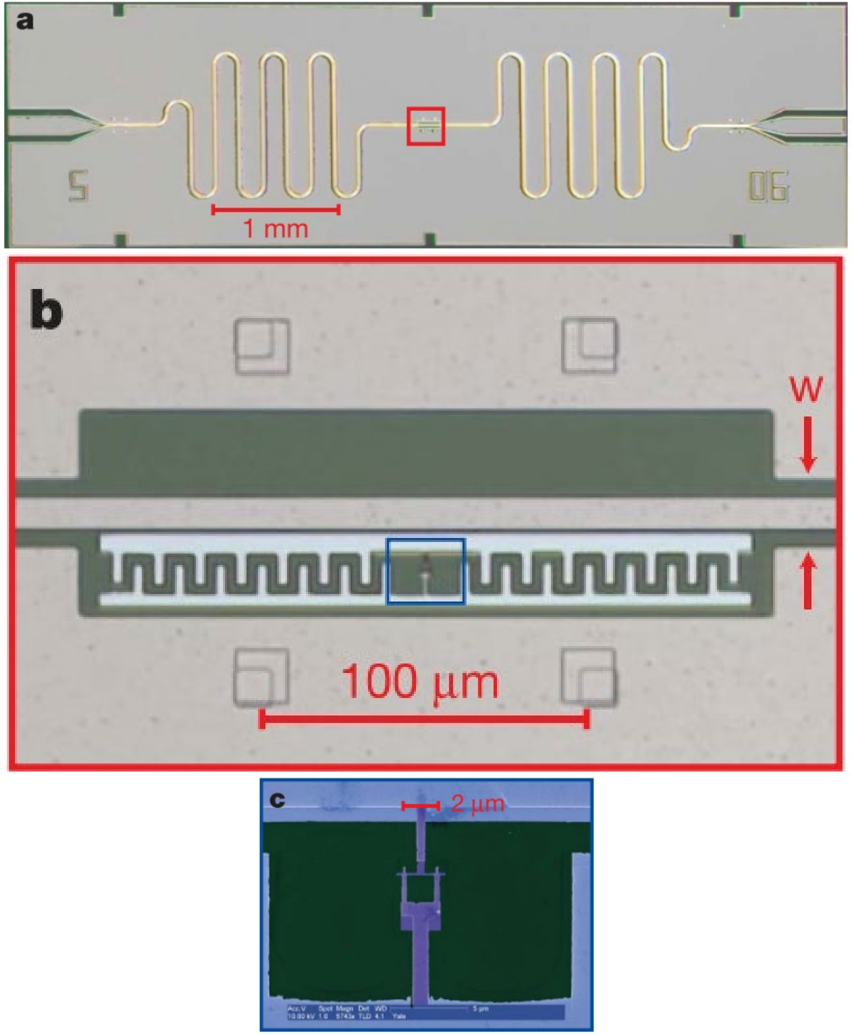
PHYS4126 – Quantum Nanoscience
Lecture 1
Superconducting quantised circuits

L1, Week 1

THE UNIVERSITY OF SYDNEY



Outline for Week 1:

Lecture Number (Week)	Planned content
1 (Week 1)	Introduction to Quantum Nanoscience, Qutip, How to read a scientific paper, Quantum harmonic oscillator revision.
2 (Week 1)	Cavity QED: Jaynes Cummings Hamiltonian
3 (Week 1)	Introducing loss, different regimes of dynamics

Introduction to Quantum Nanoscience

Canvas page:
https://canvas.sydney.edu.au/courses/49783/pages/introduction-to-quantum-nanoscience

1					2					3					4					5						
Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri		
31-Jul	1-Aug	2-Aug	3-Aug	4-Aug	7-Aug	8-Aug	9-Aug	10-Aug	11-Aug	14-Aug	15-Aug	16-Aug	17-Aug	18-Aug	21-Aug	22-Aug	23-Aug	24-Aug	25-Aug	28-Aug	29-Aug	30-Aug	31-Aug	1-Sep		
		10am SNH3003	10am SNH3003	10am SNH3003			10am SNH3003	10am SNH3003	10am SNH3003			10am SNH3003	10am SNH3003	10am SNH3003			10am SNH3003	10am SNH3003	10am SNH3003			10am SNH3003	10am SNH3003	10am SNH3003	CQED	W1 = John Bartholomew W2&3 = Kun Zuo
Module 1: SUPERCONDUCTING QUANTIZED CIRCUITS															Module 2: SPINS IN SOLIDS											
										Ass. 1 released									Ass. 1 due							

6					7					8										9						
Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri		
4-Sep	5-Sep	6-Sep	7-Sep	8-Sep	11-Sep	12-Sep	13-Sep	14-Sep	15-Sep	18-Sep	19-Sep	20-Sep	21-Sep	22-Sep	25-Sep	26-Sep	27-Sep	28-Sep	29-Sep	2-Oct	3-Oct	4-Oct	5-Oct	6-Oct		
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							REVISION			Module 3: TRAPPED IONS															IONS	Ting Rei Tan
Ass. 2 released									Ass. 2 due										Essay topic due	Ass. 3 released						

10					11					12					13											
Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri		
9-Oct	10-Oct	11-Oct	12-Oct	13-Oct	16-Oct	17-Oct	18-Oct	19-Oct	20-Oct	23-Oct	24-Oct	25-Oct	26-Oct	27-Oct	30-Oct	31-Oct	1-Nov	2-Nov	3-Nov	6-Nov	7-Nov	8-Nov	9-Nov	10-Nov		
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Module 4: NEXT GENERATION QUBITS										HDR MODERN NANOSCIENCE							REVISION								HDR	HDR
				Ass. 3 due	Ass. 4 released											Ass. 4 due			Essay due							

Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri	Mon	Tue	Wed	Thur	Fri
13-Nov	14-Nov	15-Nov	16-Nov	17-Nov	20-Nov	21-Nov	22-Nov	23-Nov	24-Nov	27-Nov	28-Nov	29-Nov	30-Nov	1-Dec	4-Dec	5-Dec	6-Dec	7-Dec	8-Dec	11-Dec	12-Dec	13-Dec	14-Dec	15-Dec
				Exams																				

Introduction to Quantum Nanoscience

Primary reading:

Module 1:
*D. I. Schuster et al., "Resolving photon number states in a superconducting circuit", Nature **445**, 515–518 (2007)*

Module 2:
*R. E. Evans et al., "Photon-mediated interactions between quantum emitters in a diamond nanocavity", Science, **362**, 662–665 (2018).*

Module 3:
V. M. Schafer et al., "*Fast Quantum Logic Gates with Trapped-Ion Qubits*", Nature **555**, 75 (2018).

Module 4:
R. M. Lutchyn et al., "Majorana zero modes in superconductor-semiconductor heterostructures", Nature Reviews Materials 3, 52 (2018).

- Download and start reading these papers now!
- This is not the only reading that will be useful for this course – keep an eye out for references in slides.

Introduction to Quantum Nanoscience

Qutip:

- Example work books for Modules 1 & 2
  https://canvas.sydney.edu.au/courses/49783/pages/qutip-jupyter-notebooks?module_item_id=1876777

- Assignments to be submitted as Qutip notebooks

How to read a scientific paper

S. Keshav, *How to Read a Paper* = 3 pass approach (https://web.stanford.edu/class/ee384m/Handouts/HowtoReadPaper.pdf)

1. 5 – 10 minutes: concentrate on the 5 C's
2. ~1 hour: concentrate on the figures
3. 4 – 5 hours: recreate the authors' results based on their assumptions

M. Mitzenmacher, *How to read a research paper* (https://www.eecs.harvard.edu/~michaelm/postscripts/ReadPaper.pdf)

- Read critically and creatively
- After the first pass, summarise the paper in one or two sentences (Due Lecture 2)

https://violentmetaphors.com/2013/08/25/how-to-read-and-understand-a-scientific-paper-2/

10 stages of reading a scientific paper:

https://www.sciencemag.org/careers/2016/01/how-read-scientific-paper

Optimism → Fear → Regret → Corner cutting → Bafflement → Distraction → Realisation → Determination → Rage → Career change

Superconducting quantised circuits

Resolving photon number states in a superconducting circuit

D. I. Schuster[1]*, A. A. Houck[1]*, J. A. Schreier[1], A. Wallraff[1]†, J. M. Gambetta[1], A. Blais[1]†, L. Frunzio[1], J. Majer[1], B. Johnson[1], M. H. Devoret[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

Electromagnetic signals are always composed of photons, although in the circuit domain those signals are carried as voltages and currents on wires, and the discreteness of the photon's energy is usually not evident. However, by coupling a superconducting quantum bit (qubit) to signals on a microwave transmission line, it is possible to construct an integrated circuit in which the presence or absence of even a single photon can have a dramatic effect. Such a system[1] can be described by circuit quantum electrodynamics (QED)—the circuit equivalent of cavity QED, where photons interact with atoms or quantum dots. Previously, circuit QED devices were shown to reach the resonant strong coupling regime, where a single qubit could absorb and re-emit a single photon many times[2]. Here we report a circuit QED experiment in the strong dispersive limit, a new regime where a single photon has a large effect on the qubit without ever being absorbed. The hallmark of this strong dispersive regime is that the qubit transition energy can be resolved into a separate spectral line for each photon number state of the microwave field. The strength of each line is a measure of the probability of finding the corresponding photon number in the cavity. This effect is used to distinguish between coherent and thermal fields, and could be used to create a photon statistics analyser. As no photons are absorbed by this process, it should be possible to generate non-classical states of light by measurement and perform qubit–photon conditional logic, the basis of a logic bus for a quantum computer.

*D. I. Schuster et al., "Resolving photon number states in a superconducting circuit", Nature **445**, 515–518 (2007)*

Introductory material to a paper

How to construct a *Nature* summary paragraph

Annotated example taken from *Nature* **435,** 114–118 (5 May 2005).

One or two sentences providing a **basic introduction** to the field, comprehensible to a scientist in any discipline.

Two to three sentences of **more detailed background**, comprehensible to scientists in related disciplines.

One sentence clearly stating the **general problem** being addressed by this particular study.

One sentence summarizing the main result (with the words "**here we show**" or their equivalent).

Two or three sentences explaining what the **main result** reveals in direct comparison to what was thought to be the case previously, or how the main result adds to previous knowledge.

One or two sentences to put the results into a more **general context**.

Two or three sentences to provide a **broader perspective**, readily comprehensible to a scientist in any discipline, may be included in the first paragraph if the editor considers that the accessibility of the paper is significantly enhanced by their inclusion. Under these circumstances, the length of the paragraph can be up to 300 words. (This example is 190 words without the final section, and 250 words with it).

During cell division, mitotic spindles are assembled by microtubule-based motor proteins[1,2]. The bipolar organization of spindles is essential for proper segregation of chromosomes, and requires plus-end-directed homotetrameric motor proteins of the widely conserved kinesin-5 (BimC) family[3]. Hypotheses for bipolar spindle formation include the 'push–pull mitotic muscle' model, in which kinesin-5 and opposing motor proteins act between overlapping microtubules[2,4,5]. However, the precise roles of kinesin-5 during this process are unknown. Here we show that the vertebrate kinesin-5 Eg5 drives the sliding of microtubules depending on their relative orientation. We found in controlled *in vitro* assays that Eg5 has the remarkable capability of simultaneously moving at ~20 nm s^{-1} towards the plus-ends of each of the two microtubules it crosslinks. For anti-parallel microtubules, this results in relative sliding at ~40 nm s^{-1}, comparable to spindle pole separation rates *in vivo*[6]. Furthermore, we found that Eg5 can tether microtubule plus-ends, suggesting an additional microtubule-binding mode for Eg5. Our results demonstrate how members of the kinesin-5 family are likely to function in mitosis, pushing apart interpolar microtubules as well as recruiting microtubules into bundles that are subsequently polarized by relative sliding. We anticipate our assay to be a starting point for more sophisticated *in vitro* models of mitotic spindles. For example, the individual and combined action of multiple mitotic motors could be tested, including minus-end-directed motors opposing Eg5 motility. Furthermore, Eg5 inhibition is a major target of anti-cancer drug development, and a well-defined and quantitative assay for motor function will be relevant for such developments.

Superconducting quantised circuits

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D. I. Schuster et al., "Resolving photon number states in a superconducting circuit", Nature **445**, 515–518 (2007)

Superconducting quantised circuits

Additional recommended reading and resources:

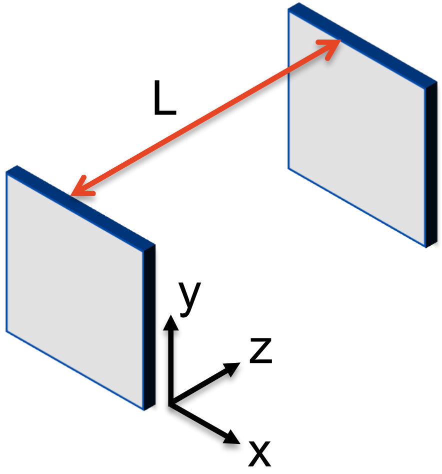
P. Krantz et al., “A Quantum Engineer’s Guide to Superconducting Qubits” https://arxiv.org/pdf/1904.06560.pdf

A. Blais et al., “Circuit quantum electrodynamics”, Reviews of Modern Physics, 93 (2021) https://journals.aps.org/rmp/pdf/10.1103/RevModPhys.93.025005

Walls and Milburn, *Quantum Optics* https://link.springer.com/book/10.1007%2F978-3-540-28574-8

Scully and Zubairy, *Quantum Optics* https://books.google.com.au/books?id=9lkgAwAAQBAJ&printsec=frontcover#v=onepage&q&f=false

Quantised electromagnetic field



Maxwell's equations in free space give rise to the homogeneous electromagnetic wave equation:

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Assume that ***E***(***r***, *t)* is polarised along **x**
& hence ***B***(***r***, *t)* is polarised along **y**

The wave equation then simplifies to:

Solutions:

$$E_x(z,t) = \sum_j \sqrt{\frac{2{\omega_j}^2 m_j}{V\varepsilon_0}} q_j(t)\sin(k_j z) \qquad H_y(z,t) = \sum_j \frac{\varepsilon_0}{k_j}\sqrt{\frac{2{\omega_j}^2 m_j}{V\varepsilon_0}} \dot{q}_j(t)\cos(k_j z)$$

Quantised electromagnetic field

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Electromagnetic field energy:

$$\begin{aligned} W &= \frac{1}{2}\int_V dV\,\left(\varepsilon_0 {E_x}^2 + \mu_0 {H_y}^2\right) \\ &= \frac{1}{2}\sum_j \left(\varepsilon_0 \frac{2{\omega_j}^2 m_j}{V\varepsilon_0} {q_j}^2(t) A \int_0^L \sin^2(k_j z)dz + \mu_0 \frac{{\varepsilon_0}^2}{{k_j}^2}\frac{2{\omega_j}^2 m_j}{V\varepsilon_0}\dot{q}_j^2(t) A \int_0^L \cos^2(k_j z)dz\right) \\ &= \frac{1}{2}\sum_j {\omega_j}^2 m_j {q_j}^2(t) + m_j \dot{q}^2(t) \end{aligned}$$

Quantised electromagnetic field

$$W = \sum_j \frac{{p_j}^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 {q_j}^2(t)}{2}$$

Energy of classical harmonic oscillator	Energy of the jth mode of the EM field in a 1D lossless cavity
$W = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$ where, $p = m\frac{dx}{dt} = m\dot{x}$	$W = \frac{{p_j}^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 {q_j}^2(t)}{2}$ where, $p_j(t) = m_j \dot{q}_j(t)$

$$\hat{H} = \sum_j \frac{\hat{p}_j^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 \hat{x}_j^2(t)}{2}$$

Quantised electromagnetic field

$$\hat{H} = \sum_j \frac{\hat{p}_j^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 \hat{x}_j^2(t)}{2}$$

$$\hat{H}_j = -\frac{\hbar m_j \omega_j}{4m_j}(\hat{a}_j^\dagger - \hat{a}_j)^2 + \frac{m_j {\omega_j}^2 \hbar}{4m_j\omega_j}(\hat{a}_j^\dagger + \hat{a}_j)^2$$

$$= \frac{\hbar\omega_j}{4}\left[(\hat{a}_j^\dagger + \hat{a}_j)^2 - (\hat{a}_j^\dagger - \hat{a}_j)^2\right]$$

$$= \frac{\hbar\omega_j}{2}(\hat{a}_j\hat{a}_j^\dagger + \hat{a}_j^\dagger\hat{a}_j)$$

$$= \hbar\omega_j(\hat{a}_j^\dagger\hat{a}_j + 1/2)$$

Quantised electromagnetic field

$$\hat{H} = \sum_j \hbar\omega_j(\hat{a}_j^\dagger \hat{a}_j + 1/2)$$

PHYS4126 – Quantum Nanoscience
Lecture 2
Superconducting quantised circuits

L2, Week 1

https://sydney.padlet.org/johnbartholomew1/PHYS4126

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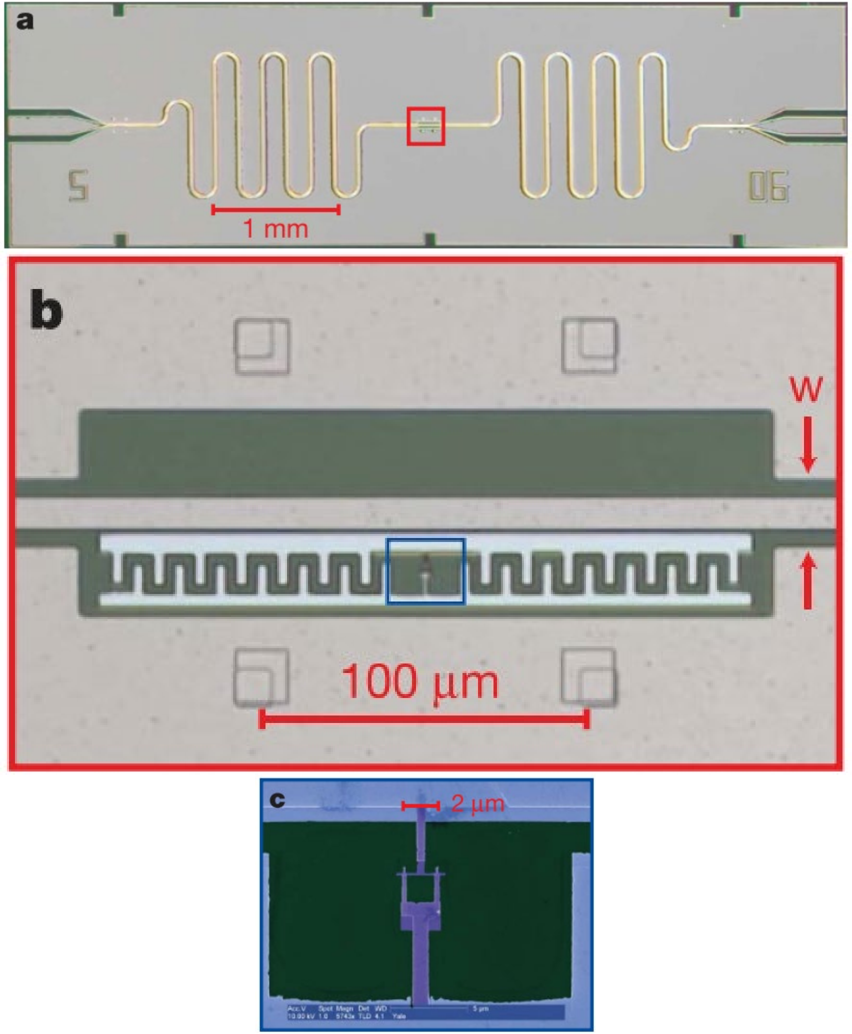

Outline for Week 1:

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1 (Week 1)	Introduction to Quantum Nanoscience, Qutip, How to read a scientific paper, Quantum harmonic oscillator revision.
2 (Week 1)	Cavity QED: Jaynes Cummings Hamiltonian
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How to read a scientific paper

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10 stages of reading a scientific paper:

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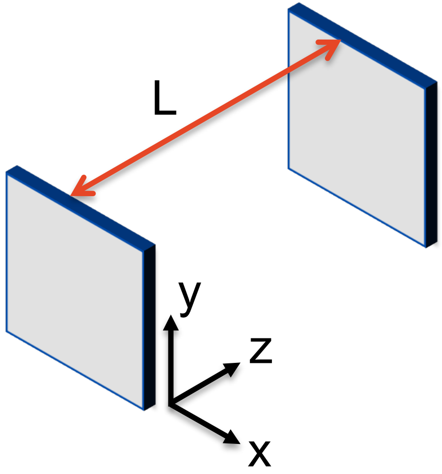
Quantised electromagnetic field



Maxwell's equations in free space give rise to the homogeneous electromagnetic wave equation:

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↓

1D wave equation

↓

Energy of the EM field

Quantised electromagnetic field

Solutions: $$E_x(z,t) = \sum_j \sqrt{\frac{2\omega_j{}^2 m_j}{V\varepsilon_0}} q_j(t)\sin(k_j z) \qquad H_y(z,t) = \sum_j \frac{\varepsilon_0}{k_j}\sqrt{\frac{2\omega_j{}^2 m_j}{V\varepsilon_0}} \dot{q}_j(t)\cos(k_j z)$$

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$$= \frac{1}{2}\sum_j \left(\varepsilon_0 \frac{2\omega_j{}^2 m_j}{V\varepsilon_0} q_j{}^2(t) A \int_0^L \sin^2(k_j z)dz + \mu_0 \frac{\varepsilon_0{}^2}{k_j{}^2}\frac{2\omega_j{}^2 m_j}{V\varepsilon_0}\dot{q}_j^2(t) A \int_0^L \cos^2(k_j z)dz\right)$$

$$= \frac{1}{2}\sum_j \omega_j{}^2 m_j q_j{}^2(t) + m_j \dot{q}^2(t)$$

Quantised electromagnetic field

$$W = \sum_j \frac{{p_j}^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 {q_j}^2(t)}{2}$$

Energy of classical harmonic oscillator	**Energy of the jth mode of the EM field in a 1D lossless cavity**
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$$\hat{H} = \sum_j \frac{\hat{p}_j^2(t)}{2m_j} + \frac{m_j {\omega_j}^2 \hat{x}_j^2(t)}{2}$$

Quantised electromagnetic field

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$$\hat{H}_j = -\frac{\hbar m_j \omega_j}{4m_j}(\hat{a}_j^\dagger - \hat{a}_j)^2 + \frac{m_j {\omega_j}^2 \hbar}{4m_j\omega_j}(\hat{a}_j^\dagger + \hat{a}_j)^2$$

$$= \frac{\hbar\omega_j}{4}\left[(\hat{a}_j^\dagger + \hat{a}_j)^2 - (\hat{a}_j^\dagger - \hat{a}_j)^2\right]$$

$$= \frac{\hbar\omega_j}{2}(\hat{a}_j\hat{a}_j^\dagger + \hat{a}_j^\dagger\hat{a}_j)$$

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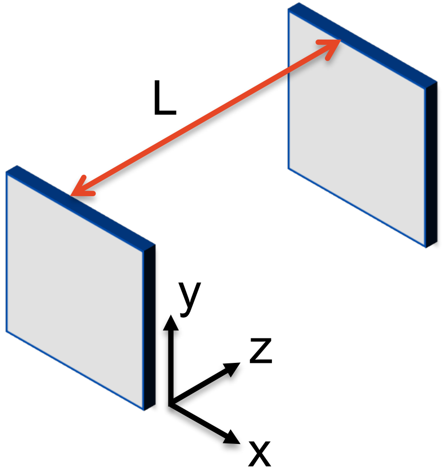
Quantised electromagnetic field

$$\hat{H} = \sum_j \hbar\omega_j(\hat{a}_j^\dagger \hat{a}_j + 1/2)$$

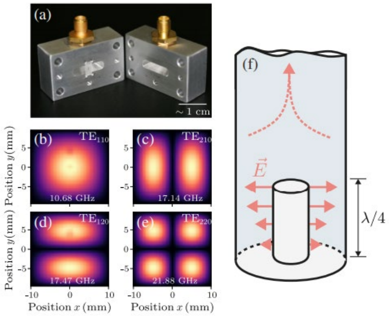
Quantised electromagnetic field

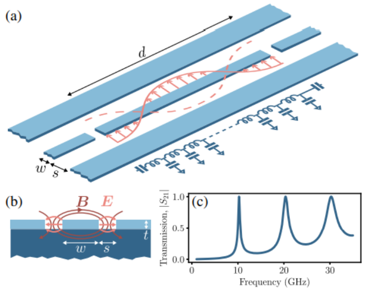


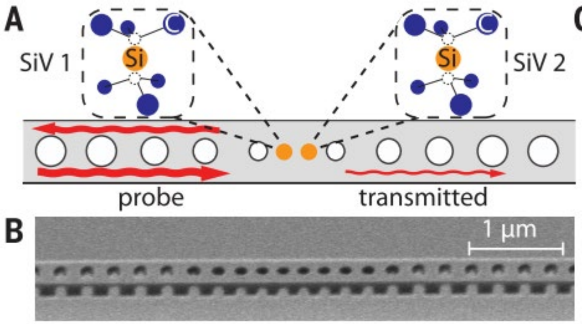
Maxwell's equations in free space give rise to the homogeneous electromagnetic wave equation:

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\hat{H} = \hbar\omega_r \hat{a}^\dagger \hat{a}$$







Quantised atom

Let's keep the model for the 'atom' simple and generic to start with

Identity: $\hat{\mathbb{I}} = |e\rangle\langle e| + |g\rangle\langle g|$

Pauli z: $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$

$$\hat{H}_a = \frac{E_0}{2}(\hat{\mathbb{I}}_a - \hat{\sigma}_z) + \frac{E_1}{2}(\hat{\mathbb{I}}_a + \hat{\sigma}_z)$$

Atom-field interaction

Briefly to atomic physics and work in the semi-classical approximation to introduce atom-field interaction.

Atom Hamiltonian is: $\hat{H}_0 = \frac{\hat{\boldsymbol{p}}^2}{2m} + V(\boldsymbol{r})$

, which becomes $\hat{H} = \frac{1}{2m}\left[(\hat{\boldsymbol{p}} + e\boldsymbol{A})^2 - e\boldsymbol{\Phi} - e\hbar g\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{B}\right] + V(\boldsymbol{r})$

in an external electromagnetic field ($H|\psi\rangle = i\hbar\frac{\partial}{\partial t}|\psi\rangle$ is the Pauli Equation).

$$\hat{H} = \frac{1}{2m}\left[(\hat{\boldsymbol{p}} + e\boldsymbol{A})^2 - e\hbar g\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{B}\right] + V(\boldsymbol{r})$$

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \frac{1}{2m}\left(e\hat{\boldsymbol{p}} \cdot \boldsymbol{A} + e\boldsymbol{A} \cdot \hat{\boldsymbol{p}} + e^2|\boldsymbol{A}|^2\right) \\ &= \hat{H}_0 + \frac{1}{2m}\left(2e\boldsymbol{A} \cdot \hat{\boldsymbol{p}} + e^2|\boldsymbol{A}|^2\right)\end{aligned}$$

Atom-field interaction

$$\hat{H} = \hat{H}_0 + \frac{1}{2m}\left(2e\boldsymbol{A}\cdot\hat{\boldsymbol{p}} + e^2|\boldsymbol{A}|^2\right)$$

Dipole approximation: electromagnetic field can be considered a plane wave at the atom.

$$\hat{H} \approx \hat{H}_0 + \frac{e}{m}\left(\boldsymbol{A}\cdot\hat{\boldsymbol{p}}\right)$$

Atom-field interaction

$$\hat{H}_{\mathrm{int},1} = \frac{e}{m}\boldsymbol{A}\cdot\hat{\boldsymbol{p}}$$

$$\hat{H}_{\mathrm{int},2} = -\hat{\boldsymbol{d}}\cdot\boldsymbol{E}$$

Note: $H_{\mathrm{int},\,1}$ and $H_{\mathrm{int},\,2}$ are equivalent in the Coulomb gauge (Radiation gauge).
Proof – see *Quantum Optics*, Scully & Zubairy, Section 5.1, 5A, p178 (see also p148-151)
– see Introductory Quantum Optics, Gerry & Knight, Section 4.1, p74

Atom-field interaction: quantum mechanically

$$\hat{H}_{\text{int},2} = q\hat{\boldsymbol{r}} \cdot \boldsymbol{E}$$

Identity: $\hat{\mathbb{I}} = |e\rangle\langle e| + |g\rangle\langle g|$

$$q\hat{\boldsymbol{r}} = \hat{\mathbb{I}}(q\hat{\boldsymbol{r}})\hat{\mathbb{I}} = (|e\rangle\langle e| + |g\rangle\langle g|)(q\hat{\boldsymbol{r}})(|e\rangle\langle e| + |g\rangle\langle g|)$$

Define $\boldsymbol{\mu}_{ij} = \langle i|q\hat{\boldsymbol{r}}|j\rangle$ and operators $\hat{\sigma}_{ij} = |i\rangle\langle j|$

$$q\boldsymbol{r} = \boldsymbol{\mu}_{eg}\hat{\sigma}_{eg} + \boldsymbol{\mu}^*_{eg}\hat{\sigma}_{ge} \equiv \boldsymbol{\mu}_{eg}\hat{\sigma}_{+} + \boldsymbol{\mu}^*_{eg}\hat{\sigma}_{-}$$

What about ***E*** ?

Atom-field interaction: quantum mechanically

$$\hat{H}_{\text{int},2} = -q\hat{\boldsymbol{r}} \cdot \boldsymbol{E}$$

$$\hat{H}_{\text{int},2} = \left(\boldsymbol{\mu}_{eg}\hat{\sigma}_{+} + \boldsymbol{\mu}^{*}_{eg}\hat{\sigma}_{-}\right) \cdot \hbar\boldsymbol{E_0}(\hat{a}^{\dagger} + \hat{a})$$

$$\hat{H}_{\text{int},2} = \hbar g\left(\hat{\sigma}_{+}\hat{a} + \hat{\sigma}_{-}\hat{a}^{\dagger}\right)$$

Jaynes Cummings Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right)$$

Exercise:

What is the key message of this figure?
Define the important terms and parameters.

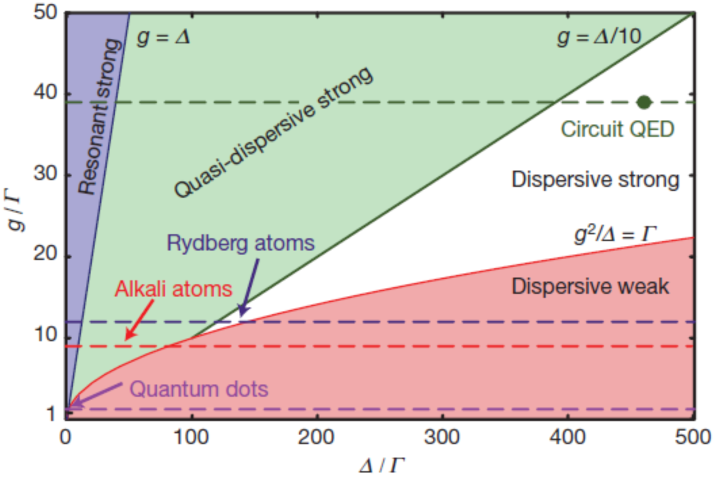
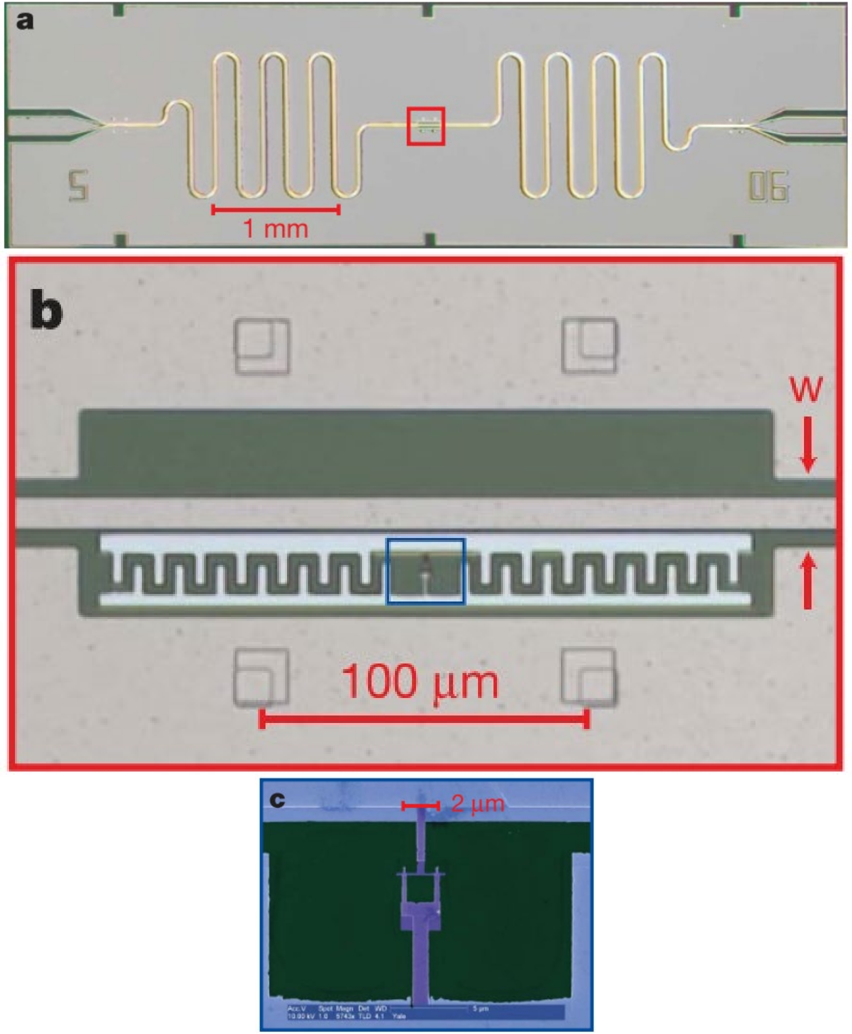


Figure 1 | A parameter space diagram for cavity QED. The space is described by the atom–photon coupling strength, g, and the detuning Δ between the atom and cavity frequencies, normalized to the rates of decay represented by $\Gamma = \max[\gamma, \kappa, 1/T]$. Different cavity QED systems, including Rydberg atoms, alkali atoms, quantum dots, and circuit QED, are represented by dashed horizontal lines. The dark green filled circle represents the parameters used in this work. In the blue region the qubit and cavity are resonant, and undergo vacuum Rabi oscillations. In the red, weak dispersive, region the a.c. Stark shift $g^2/\Delta < \Gamma$ is too small to dispersively resolve individual photons, but a QND measurement of the qubit can still be realized by using many photons. In the white region QND measurements are in principle possible with demolition less than 1%, allowing 100 repeated measurements. In the green region single photon resolution is possible but measurements of either the qubit or cavity occupation cause larger demolition.

PHYSx126 – Quantum Nanoscience
Lecture 3
Superconducting quantised circuits

L3, Week 1



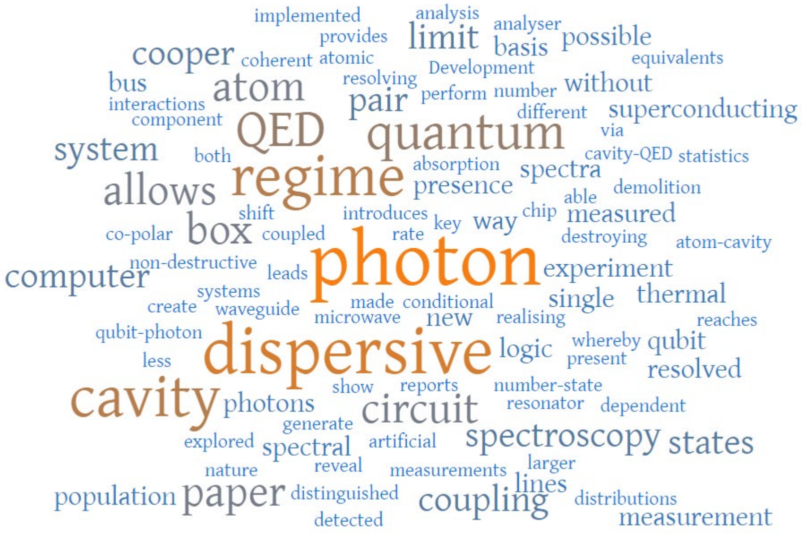
THE UNIVERSITY OF SYDNEY

Outline for Week 1:

Lecture Number (Week)	Planned content
1 (Week 1)	Introduction to Quantum Nanoscience, Qutip, How to read a scientific paper, Quantum harmonic oscillator revision.
2 (Week 1)	Cavity QED: Jaynes Cummings Hamiltonian
3 (Week 1)	Introducing loss, different regimes of dynamics

2 Sentence summaries:

2021



2022

Atom-field interaction

$$\hat{H}_{\mathrm{int},1} = \frac{e}{m}\boldsymbol{A}\cdot\hat{\boldsymbol{p}}$$

$$\hat{H}_{\mathrm{int},2} = -\hat{\boldsymbol{d}}\cdot\boldsymbol{E}$$

Note: $H_{\text{int, 1}}$ and $H_{\text{int, 2}}$ are equivalent in the Coulomb gauge (Radiation gauge).
Proof – see *Quantum Optics*, Scully & Zubairy, Section 5.1, 5A, p178 (see also p148-151)
– see Introductory Quantum Optics, Gerry & Knight, Section 4.1, p74

Atom-field interaction: quantum mechanically

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What about ***E*** ?

Atom-field interaction: quantum mechanically

$$\hat{H}_{\mathrm{int},2} = q\boldsymbol{r} \cdot \boldsymbol{E}$$

$$\hat{H}_{\mathrm{int},2} = \left(\boldsymbol{\mu}_{eg}\hat{\sigma}_{+} + \boldsymbol{\mu}_{eg}^{*}\hat{\sigma}_{-}\right) \cdot \hbar\boldsymbol{E}_{\mathbf{0}}(\hat{a}^{\dagger} + \hat{a})$$

$$\hat{H}_{\mathrm{int},2} = \hbar g \left(\hat{\sigma}_{+}\hat{a} + \hat{\sigma}_{-}\hat{a}^{\dagger}\right)$$

Atom-field interaction: quantum mechanically

$$\hat{H}_{\text{int},2} = q\boldsymbol{r} \cdot \boldsymbol{E}$$

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Jaynes Cummings Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right)$$

Edwin Jaynes

Frederick Cummings

Jaynes Cummings Hamiltonian

$$H_{\mathrm{JC}} = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right)$$

What assumptions have we made thus far?

Adding loss to the Jaynes Cummings Hamiltonian

Cavity field decay rate: $\kappa = \frac{\omega_r}{2Q}$

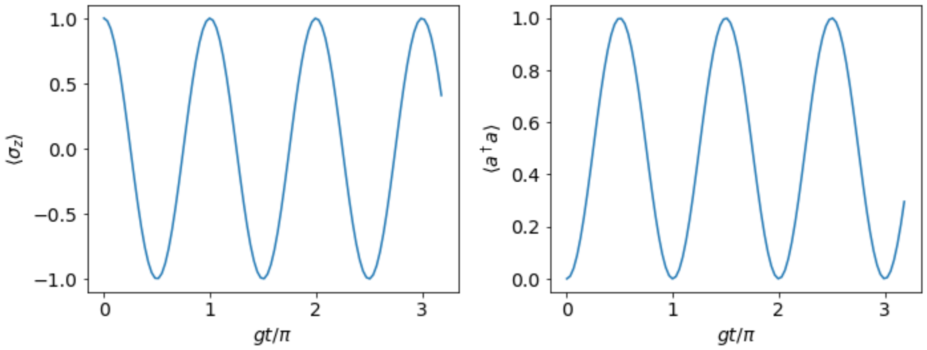
Atom decay rate: γ

$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right) + H_{Loss,r} + H_{Loss,a}$$

Time dynamics when $\kappa = \gamma = 0$

$$H_{\mathrm{JC}} = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g \left(a\sigma_+ + a^\dagger \hat{\sigma}_-\right)$$

Jupyter notebook #1: $\omega_a = \omega_r = 10, \qquad g = 1$



Time dynamics when $\kappa, \gamma \neq 0$

Jupyter notebook #2 & Extra notebook #1

$$H = \frac{\hbar\omega_a}{2}\sigma_z + H_{Loss,a}$$

Different regimes of the JC Hamiltonian

$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right) + H_{Loss,r} + H_{Loss,a}$$

<u>Strong coupling regime</u>

Condition: $g \gg \kappa, \gamma$

In this case, the system Hamiltonian can be approximated by the Jaynes Cummings Hamiltonian.
The coherent coupling rate between the cavity mode and the 'atom' is large compared to the (irreversible) dissipation rates.

Rabi oscillations occur:

Different regimes of the JC Hamiltonian

$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right) + H_{Loss,r} + H_{Loss,a}$$

Weak coupling regime

Condition: $g < \kappa, \gamma$

In this case, the (irreversible) dissipation rates dominate the coherent coupling.
An interesting way to consider this is that the atom interacts with many fields. The resulting Rabi oscillations are all at different frequencies, which leads to destructive interference of probability amplitudes corresponding to the different interactions. The result is irreversible spontaneous emission.

Different regimes of the JC Hamiltonian

$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right) + H_{Loss,r} + H_{Loss,a}$$

Resonant regime

Condition: $\omega_a \approx \omega_r$

Dispersive regime

Condition: $|\omega_a - \omega_r| = |\Delta| \gg g$

Different regimes of the JC Hamiltonian

$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\hat{\sigma}_-\right) + H_{Loss,r} + H_{Loss,a}$$

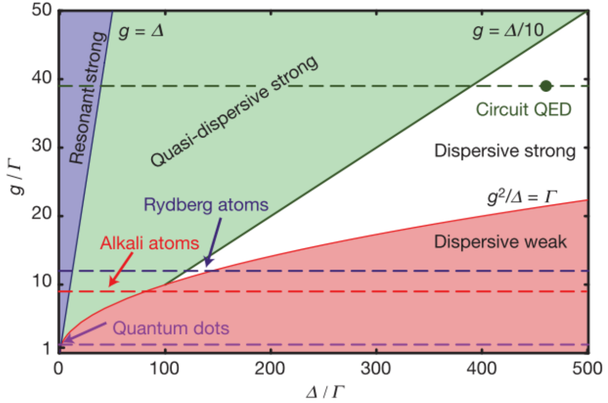


Figure 1 | A parameter space diagram for cavity QED. The space is described by the atom–photon coupling strength, g, and the detuning Δ between the atom and cavity frequencies, normalized to the rates of decay represented by $\Gamma = \max[\gamma, \kappa, 1/T]$. Different cavity QED systems, including Rydberg atoms, alkali atoms, quantum dots, and circuit QED, are represented by dashed horizontal lines. The dark green filled circle represents the parameters used in this work. In the blue region the qubit and cavity are resonant, and undergo vacuum Rabi oscillations. In the red, weak dispersive, region the a.c. Stark shift $g^2/\Delta < \Gamma$ is too small to dispersively resolve individual photons, but a QND measurement of the qubit can still be realized by using many photons. In the white region QND measurements are in principle possible with demolition less than 1%, allowing 100 repeated measurements. In the green region single photon resolution is possible but measurements of either the qubit or cavity occupation cause larger demolition.

Exercise:

What is the key message of this figure?
Define the important terms and parameters.

Primer on unitary transformations

$$|\psi_u\rangle = U^\dagger|\psi\rangle$$

$$\frac{d}{dt}|\psi\rangle = \frac{-i}{\hbar}H|\psi\rangle$$

$$\frac{d}{dt}U|\psi_u\rangle = \frac{-i}{\hbar}HU|\psi_u\rangle$$

$$\dot{U}|\psi_u\rangle + U\frac{d}{dt}|\psi_u\rangle = \frac{-i}{\hbar}HU|\psi_u\rangle$$

$$\frac{d}{dt}|\psi_u\rangle = U^\dagger\left(\frac{-i}{\hbar}HU - \dot{U}\right)|\psi_u\rangle$$

$$\frac{d}{dt}|\psi_u\rangle = \frac{-i}{\hbar}\left(U^\dagger HU - i\hbar U^\dagger\dot{U}\right)|\psi_u\rangle$$

Dispersive limit of the JC model

Useful formula (Corollary of the Baker – Campbell – Hausdorff Theorem)

$$e^S H e^{-S} = H + [S, H] + \frac{1}{2!}[S, [S, H]] + \frac{1}{3!}[S, [S, [S, H]]] + \dots$$

$$H_{\mathrm{JC}} = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g \left(a\sigma_+ + a^\dagger \hat{\sigma}_-\right)$$

Exercise: Show that by choosing $\alpha = \frac{g}{\Delta}$:

$$H_u = U^\dagger H U = \omega_r a^\dagger a + \frac{\omega_a + \chi}{2}\sigma_z + \chi a^\dagger a \sigma_z$$

Dispersive limit of the JC model

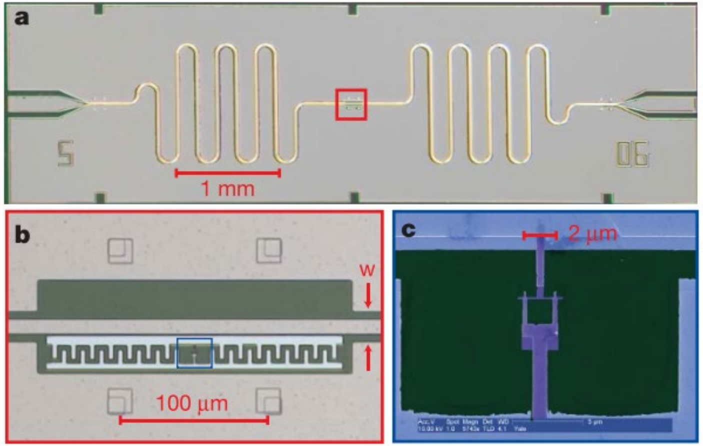
Exercise: Show that by choosing $\alpha = \frac{g}{\Delta}$:

$$H_u = U^\dagger H U = \omega_r a^\dagger a + \frac{\omega_a + \chi}{2}\sigma_z + \chi a^\dagger a \sigma_z$$

In the dispersive (off-resonant) limit, the atom cavity detuning is larger than the coupling, $\Delta \gg g$, and only virtual photon exchange is allowed, keeping the atom and photon largely separable (red and white regions in Fig. 1). The atom (photon) now acquires only a small photonic (atomic) component of magnitude $(g/\Delta)^2$, and an accompanying frequency shift, $2\chi = 2g^2/\Delta$. In this case, the dispersive and rotating wave approximations apply, and the system is described to second order in g/Δ by the quantum version of the a.c. Stark hamiltonian[1]:

$$H = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\omega_a\sigma_z/2 + \hbar\chi(a^\dagger a + 1/2)\sigma_z$$

For next lecture:



Lecturer: Dr Kun Zuo